

FactsWise + -

Adding & Subtracting with
Fluency, Flexibility & Number Sense



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NINE GOALS FOR BASIC FACTS SUCCESS

Once a child is fluent with the addition facts in each goal, begin work on the related subtraction facts.

Goal 1 - Within 4s & 5s

Add: $1+3$, $2+2$, $3+1$, $1+4$, $2+3$, $3+2$, $4+1$

Sub: $4-1$, $4-2$, $4-3$, $5-1$, $5-2$, $5-3$, $5-4$

Goal 2 - With 5s (part 1)

Add: $1+5$, $2+5$, $3+5$, $4+5$, $5+5$

Sub: $6-1$, $6-5$, $7-2$, $7-5$, $8-3$, $8-5$, $9-4$, $9-5$, $10-5$

Goal 3 - Within 10s

Add: $0+10$, $1+9$, $2+8$, $3+7$, $4+6$

Sub: $10-0$, $10-10$, $10-1$, $10-9$, $10-2$, $10-8$, $10-3$, $10-7$, $10-4$, $10-6$

Goal 4 - With 10s

Add: $10+1$, $10+2$, $10+3$, $10+4$, $10+5$, $10+6$, $10+7$, $10+8$, $10+9$, $10+10$

Sub: $11-1$, $11-10$, $12-2$, $12-10$, $13-3$, $13-10$, $14-4$, $14-10$, $15-5$, $15-10$,
 $16-6$, $16-10$, $17-7$, $17-10$, $18-8$, $18-10$, $19-9$, $19-10$, $20-10$

Goal 5 - With 5s (part 2)

Add: $5+6$, $5+7$, $5+8$, $5+9$

Sub: $11-5$, $11-6$, $12-5$, $12-7$, $13-5$, $13-8$, $14-5$, $14-9$

Goal 6 - Doubles

Add: $3+3$, $4+4$, $6+6$, $7+7$, $8+8$, $9+9$

Sub: $6-3$, $8-4$, $12-6$, $14-7$, $16-8$, $18-9$

Goal 7 - Under Tens

Add: $2+4$, $2+6$, $2+7$, $3+4$, $3+6$

Sub: $6-2$, $6-4$, $8-2$, $8-6$, $9-2$, $9-7$, $7-3$, $7-4$, $9-3$, $9-6$

Goal 8 - With 9s

Add: $2+9$, $3+9$, $4+9$, $6+9$, $7+9$, $8+9$

Sub: $11-2$, $11-9$, $12-3$, $12-9$, $13-4$, $13-9$, $15-6$, $15-9$, $16-7$, $16-9$, $17-8$, $17-9$

Goal 9 - With 7s & 8s

Add: $4+7$, $6+7$, $3+8$, $4+8$, $6+8$, $7+8$

Sub: $11-4$, $11-7$, $13-6$, $13-7$, $11-3$, $11-8$, $12-4$, $12-8$, $14-6$, $14-8$, $15-7$, $15-8$

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SECTION ONE: INTRODUCTION

We know from research that the vast majority of first-grade students from some countries (e.g., Japan, China, Korea) are able to develop a fluency with their basic facts that many U.S. students never acquire. Their fluency is built on more than simply memorization. These students seem to progress from counting to part-whole thinking while simultaneously working on memorization. Their sense of numbers develops as they take numbers apart and put them back together again.

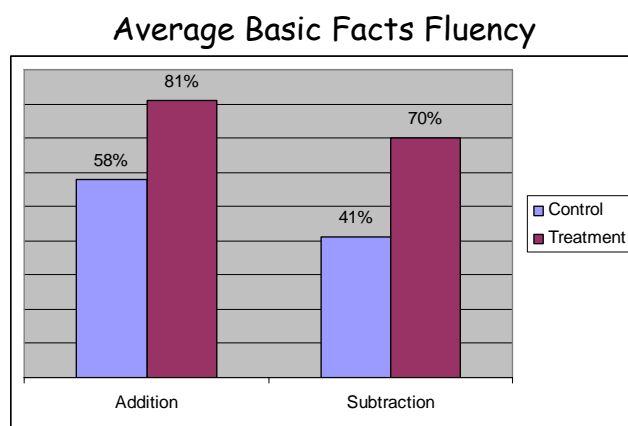
Some U.S. students develop this same fluency with numbers - but many don't. FactsWise provides a systematic approach to basic facts fluency for all our students. It builds on the research from many countries, including China, Japan, Korea, Singapore, Australia, and New Zealand. In addition, it incorporates cognitive research on ways to move facts into long-term memory, and then to help build strong retrieval mechanisms.

As we all know, memorizing a large set of facts works best when we break it up into smaller chunks. FactsWise breaks the job of memorizing the addition and subtraction facts up into nine goals. These goals are strategic - they work on fives and tens early in the program, so they can be used as tools with the remaining facts. We also know that students find subtraction much more cognitively challenging than addition, particularly when they don't see the connections between them. So FactsWise incorporates the related subtraction facts right after each addition goal.

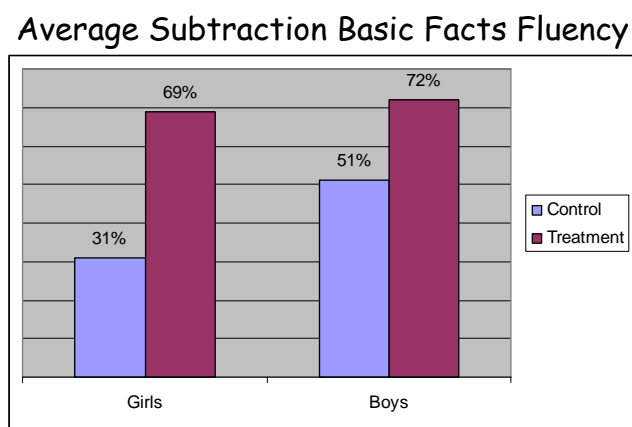
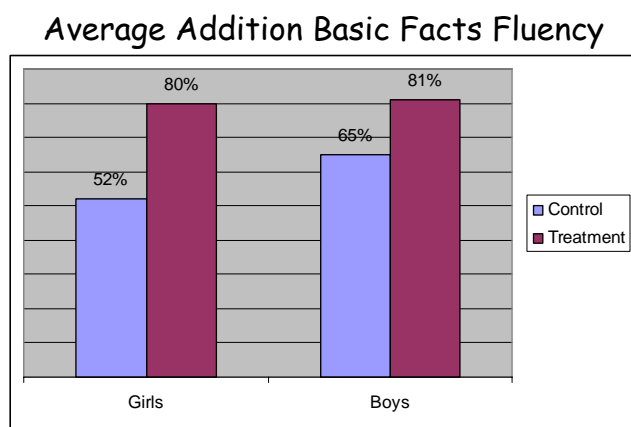
This program is different in one other important aspect - no timed tests! We all know how much anxiety timed tests can create in our classrooms. But equally troublesome, timed tests don't really tell us much about how students are solving their facts problems. If we want to help students move beyond counting, we need a different assessment tool. FactsWise provides an easy one-on-one assessment system that can be implemented by the classroom teacher, classroom aids, or parent volunteers. Teachers who have used this program are sold on what they can learn about a student's math thinking in just a minute or two of one-on-one assessment.

1.1: Research Findings

The results of this program are remarkable! The average basic facts fluency (memorized or part-whole strategy in 3 seconds or less) was significantly higher for FactsWise first-graders compared to "control" students who experienced other basic facts approaches.¹

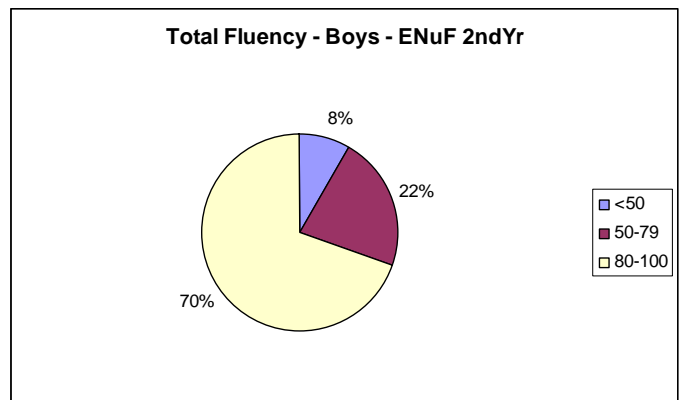
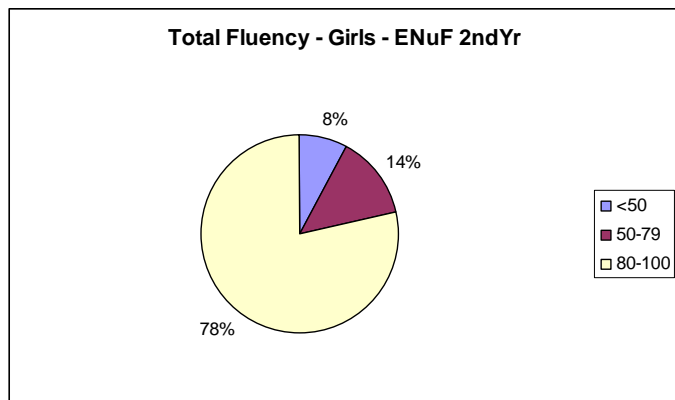
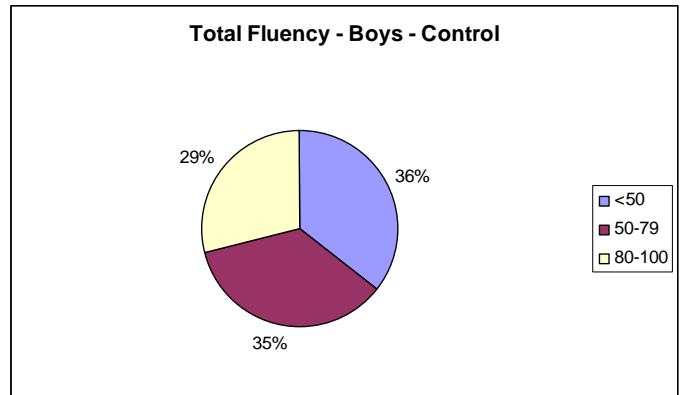
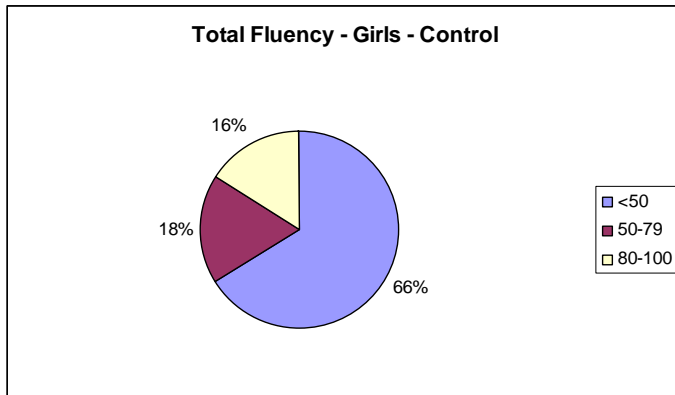


Girls in the control classrooms were significantly less fluent in both addition and subtraction. This disparity was virtually eliminated in FactsWise classrooms.



¹ One-on-one basic fact assessments were conducted in June, 2007 with 141 students from 14 FactsWise first-grade classrooms and 98 students from 10 "control" classrooms in a high-performing California school district. Assessed students were randomly selected from each classroom.

Results were even more pronounced in classrooms with teachers who were implementing FactsWise for the second year. As the yellow portions of the graphs below show, only 16% of the girls and 29% of the boys in the control classrooms demonstrated mastery of basic facts (80% or more memorized or correctly solved using part-whole thinking in 3 seconds or less). In classrooms with teachers implementing FactsWise for the second year, 78% of the girls and 70% of the boys demonstrated mastery.



Yellow - 80-100% fluent; Red - 50-79% fluent; Blue - less than 50% fluent

SECTION TWO: KEY PRINCIPLES AND STRATEGIES

2.1 Basic Facts for Number Sense

It turns out that the way a child learns to add and subtract small numbers up to $9+9$ and $18-9$ is likely to set the stage for how that child makes sense of mathematics for years to come.

All children begin to make sense of addition and subtraction by counting with objects and their fingers. This is an important first step towards making sense of numbers. In fact, counting seems to be to mathematics what phonemic awareness is to reading.

At some point, though, some children begin to take numbers apart and put them back together in ways that allow them to add and subtract without counting. For example, $9 + 7$ can be thought of as $9 + (1 + 6)$ which allows the child to make a ten with six more: $(9 + 1) + 6 = 10 + 6 = 16$. This type of part-whole thinking is an exceptionally important step forward in a child's mathematical career! Research has found a strong link between part-whole thinking and number sense in general. Children who use part-whole thinking to solve addition and subtraction problems have a big advantage over children who continue to rely solely on counting.

In the past, we have thought of speed and accuracy as the twin goals of basic facts instruction. If a student could answer basic fact problems correctly and quickly, we have often overlooked the way in which the problems were solved - particularly if fingers were out of sight! In fact, students are often very proud when they can count mentally, and don't need their fingers at all. Yet they are still counting. When students continually use counting as their primary method for solving problems, they generally aren't looking at the relationships between numbers. Even when they are as quick (or even quicker) at solving a problem than part-whole thinkers, they are missing opportunities to develop an important key to mathematical power - number sense.

Eventually, of course, we hope that students will memorize their facts. The ability to retrieve basic facts from long-term memory without needing to count or use part-whole strategies frees up working memory. This becomes very important in later years when multi-step mathematical problems tax working memory limits. What is becoming increasingly clear, though, is the long-term difference between students who practice their way to memorization through rote counting versus students who gain part-whole strategies and number sense relationships on their road to memorization.

Based on new research, we now know just how important an impact primary teachers can have on their students' long-term mathematical power. In reading, primary teachers have learned to simultaneously teach for accuracy, fluency, and comprehension. Now, in basic facts development, primary teachers have a similar challenge - to teach simultaneously for accuracy, fluency, **and** part-whole thinking.

2.2 Part-Whole Thinking and the Power of Tens

There are three main kinds of part-whole strategies for addition: making tens, working with fives, and relating to known facts.

Many U.S. teachers have focused their part-whole instruction on known-fact strategies such as doubles plus and minus one. Doubles part-whole thinking has short-term value in that it helps students solve some of the more challenging basic facts, including $5+6$, $6+7$, $7+8$, and $8+9$. It has two limitations, though. First, this strategy does not easily translate to other basic fact problems such as $6+9$ and $5+8$. Thus, teachers often attempt to teach their students several strategies to solve the complete range of over-ten basic fact problems, including doubles strategies, the nines rule, two-aparts, the eights rule, ... With so many strategies and "tricks" to remember, many students just seem overwhelmed and continue to rely on counting to solve their facts.

A second limitation to focusing on known-fact strategies is that they have limited long-term value in solving multi-digit problems mentally (e.g., $27+9$ or $35+18$). Very few multi-digit problems will fall into the doubles-plus-one or two-aparts categories.

A more powerful part-whole strategy for addition is making tens. This is the strategy many Asian students learn for over-ten facts. Once students have memorized their combinations of 10 ($1+9$, $2+8$, $3+7$, $4+6$, $5+5$), they are ready to make tens. For example:

$$\begin{array}{c} 9 + 6 \\ \swarrow \quad \searrow \\ 9 + 1 + 5 \\ \swarrow \quad \searrow \\ 10 + 5 = 15 \end{array}$$

and

$$\begin{array}{c} 8 + 5 \\ \swarrow \quad \searrow \\ 8 + 2 + 3 \\ \swarrow \quad \searrow \\ 10 + 3 = 13 \end{array}$$

The ability to make tens is a powerful mental math strategy that is extremely useful as students move into higher levels of mathematics. When students are done with the nine goals of this program, they will be ready to learn more advanced part-whole thinking to solve problems such as:

$$\begin{array}{c} 29 + 16 \\ \swarrow \quad \searrow \\ 29 + 1 + 15 \\ \swarrow \quad \searrow \\ 30 + 15 = 45 \end{array}$$

and

$$\begin{array}{c} 38 + 45 \\ \swarrow \quad \searrow \\ 38 + 2 + 43 \\ \swarrow \quad \searrow \\ 40 + 43 = 83 \end{array}$$

2.3 Subtraction

Most students find subtraction even more challenging than addition. Perhaps because of this, many instructional programs wait until students have achieved success with all of the addition facts before tackling the subtraction facts. For many students, this means they actually have less time during the school year to learn about the facts that are more difficult for them.

This program takes a different approach. Students start with a small chunk of addition facts (1+3, 2+2, 1+4, 2+3). Once they have memorized these facts, they are immediately asked to begin memorizing the related subtraction facts. For each new goal, this same approach provides students with a stable addition foundation on which to develop subtraction confidence.

Once students have completed memorizing the facts through Goal 4, they have the tools to use a part-whole subtraction strategy to solve larger subtraction fact problems. By subtracting from 10 first, students can advance beyond the cognitively demanding method of counting back to solve larger subtraction facts.

Subtracting from 10 First

$$\begin{array}{r} 15 - 8 \\ \swarrow \quad \searrow \\ 5 + 10 - 8 \\ \swarrow \quad \searrow \\ 5 + 2 = 7 \end{array} \quad \text{or} \quad \begin{array}{r} 15 \longrightarrow 10 + 5 \\ - 8 \quad \quad \quad - 8 \\ \hline \quad \quad \quad 2 + 5 = 7 \end{array}$$

Try some for yourself, using both of the recording methods above:

1) $12 - 9$

2) $13 - 4$

3) $11 - 7$

4) $15 - 6$

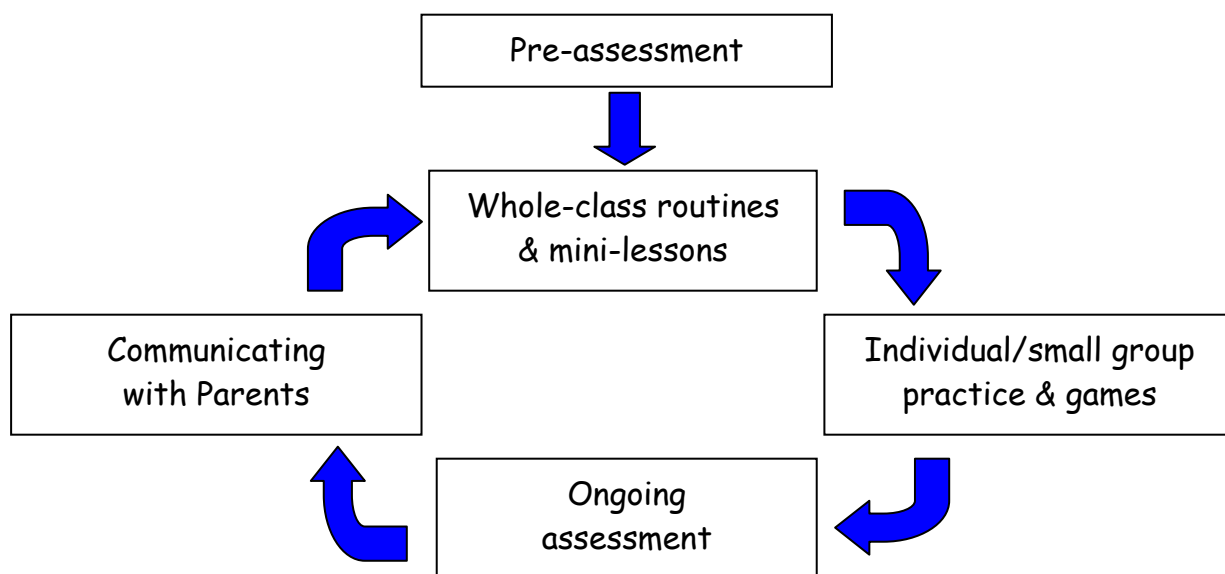
2.4 A Special Note about Doubles

Because doubles seem to be more easily memorized than other facts, many teachers have placed an early emphasis on doubles and doubles plus/minus one and two strategies in their classrooms. You may have noticed that doubles are not addressed until Goal 6 in this program. This is primarily because we want to provide students with the tens tools early on, to maximize the extremely powerful tens strategies for both addition and subtraction. When students get to the doubles in Goal 6, they are often pleased to notice how quickly they progress through this goal - at least for addition. It is quite striking how many students who find the addition doubles facts relatively easy need time to make the connections to the doubles subtraction facts.

SECTION TWO: GETTING STARTED

3.1: Early Number Fluency Overview

FactsWise is a very flexible program. You can start off very simply, and add instructional strategies, materials, and technology as you choose. The core of the program revolves around: 1) quick one-on-one assessments, 2) whole-class routines and mini-lessons, and 3) goal-specific practice and games.



Before you begin this program, be sure that your students have developed a good conceptual understanding of both addition and subtraction. It is very important that students not work on memorizing math facts before they understand what the operations mean.

FactsWise has three distinct stages:

- Goals 1-4 - continue developing conceptual understanding, particularly making connections between addition and subtraction. As you work with patterns and ten-based visual relationships, you will **focus on memorization** of these key facts.
- Goals 5-7 - focus on **developing ten-based part-whole strategies**. This part-whole thinking has long-term value for multi-digit mental math. Ultimately, students will benefit from having these facts **memorized**.
- Goals 8-9 - **focus on part-whole thinking** with these facts. As long as students have efficient part-whole strategies for these facts, it does not seem critical that they memorize all of these, although memorization is definitely optimal.

Be sure to have students do cumulative reviews after Goals 3, 6, and 9. Teachers have reported that when these are skipped, students tend to forget earlier goals.

3.2 Pre-Assessing Your Students

To get started, you have two good options:

- 1) Start everyone at Goal 1 addition and be ready for some students to move quickly through one or more goals.
- 2) Do a quick-start [FactsWise Pre-Assessment](#) (Section 8) with each student to gain a good sense of which goal each student is ready to work on. This generally takes 3 to 5 minutes per student.

3.3 Pre-Assessment and Ongoing Assessment Techniques

When you do one-on-one assessments with your students, you will want to have made some decisions ahead of time:

- 1) Will you be asking the students the questions verbally? Will you also be showing the students the problems in writing (see [FactsWise Pre-Assessment Visuals](#))? Or will you simply be asking your students to look at the problems in writing? We have found a great deal of success using verbal prompts while also showing the students the problems in writing. If, however, you have second-language students who have learned their numbers and at least some of their facts in their first language, you may want to avoid saying the problems aloud in English. For these students, this necessitates that they then translate them into their first language, and then back again when they state the answers. You may get a truer sense of their fluency if they don't have to make that first translation. Be aware that the need to translate their answers into English may still cause these students to have a certain time lag.
- 2) What are your criteria for passing students on a goal?
 - a. If a student is counting on one or more problems, then he or she is not ready to move on yet. Sometimes you'll be able to see the student using fingers or sub-vocalizing the counting.
 - b. In all cases where you're not sure what a student is doing, ask "How did you get that?" Most students will be able and willing to tell you. If they say "I just knew it", it's most likely that they are retrieving from long term memory. If they say they were counting in their heads, they're still not ready to move on. Remember, our goal is to help students develop part-whole strategies and ultimately memorization!
 - c. If a student is retrieving from long-term memory, or using a part-whole strategy, you have one last decision to make - was it fluent enough? Your criteria for fluency (speed of response) may differ depending on the grade you're working with.

- i. For second grade and above, we recommend setting a standard of two seconds or less for response time (just mentally count "one thousand one, one thousand two" while you're waiting for the response). If the student is taking longer than this, then their retrieval or part-whole strategy is still cognitively demanding.
- ii. For first grade and kindergarten, you may want to set a more forgiving fluency expectation - perhaps three seconds. One thing we definitely want to encourage at this age is part-whole thinking. If we demand that they answer too quickly during this developmental phase, we may simply encourage guessing.

3.4 Pre-Assessment and Ongoing Assessment Record Keeping

You'll find a couple of different recording options in the [Record-Keeping Resources \(Section 8\)](#).

- 1) If you do the quick-start pre-assessment, you'll want copies of the [FactsWise Pre-Assessment form](#) for all of your students.
- 2) Once you have started the program, and are conducting weekly or bi-weekly ongoing assessments, we recommend that you use individual student records ([FactsWise Progress Chart](#)). These allow you to notice students who are not making steady progress, and also provide good information for parent conferences.
- 3) In addition, we have provided a [FactsWise Class Progress Chart](#) that can provide you with flexible grouping options for small-group instruction and for FactsWise stations. You can write students' initials or class numbers on small dots with removable adhesive (available at most office supply stores). These dots are able to move from goal to goal as students progress. We generally recommend that this chart be reserved for the teacher's eyes only.

The complete 93-page book can be ordered from www.ellipsismath.com